Basic Equations—Flat Earth, AWGN

$$P_r = \frac{P_t G_t G_r h_t^2 h_r^2}{d^4} \qquad No = kTFW$$

$$P(e) = kQ(c\sqrt{Eb / No})$$

To double the range, must either:

12 dB extra antenna gain

Quadruple transmit or receive elevation

Cut Tx rate by factor of 16

Improve noise figure by 12 dB

Trades with Coding: AWGN

$$C = W \log(1 + P / NW)$$

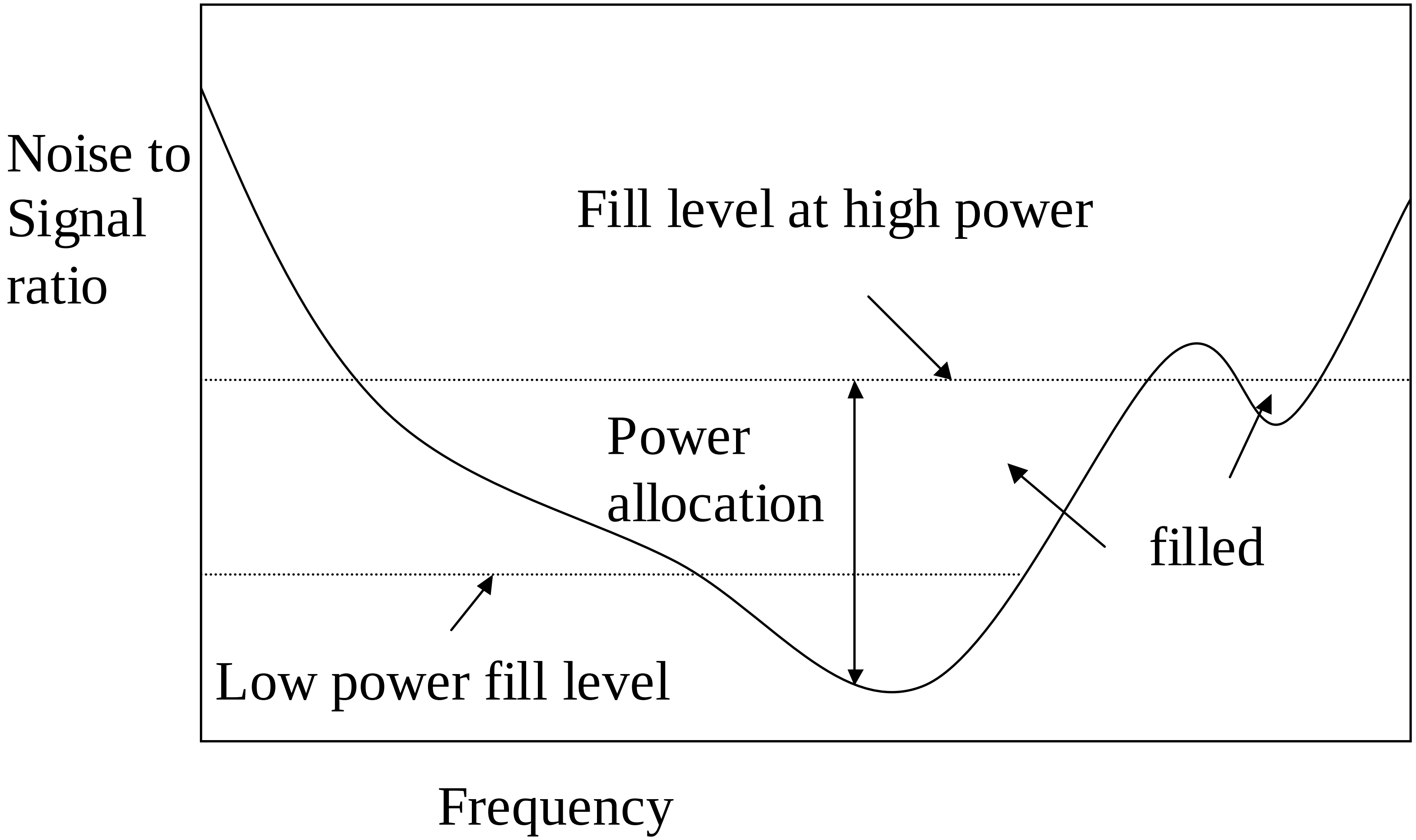
$$P = NW(10^{C/W} - 1)$$

Power efficiency improves with bandwidth.
Power declines exponentially with rate if use optimal low-rate codes.
Note: simple spread spectrum does not actually change W in above formulas.

Low Rate Signaling in Fading

- Frequency selective fading offers set of parallel channels with different SNRs to sender
- Capacity achieving solution: waterfilling on noise to signal ratio profile; allocate power where SNR is high (presumes use of heavy channel coding)
- Simpler: allocate power to “channels” with SNR above target threshold
- Opportunities for range extension compared to non-fading channel

Waterfilling Distribution



Noise to Signal ratio

Fill level at high power

Power allocation

filled

Low power fill level

Frequency

Example: Lognormal fading

- SNR (dB) follows Gaussian distribution
- If $\sigma = 8$ dB, 6% of frequencies can yield 16 dB improved SNR over average—can more than double range with fourth power loss
- If use one part in 10^6, get 38 dB improvement, or factor of 8 in range
- Also get benefit of slower signaling e.g., if drop factor of 10^6, 60 dB SNR improvement, or factor 32 in range for fourth power loss.

Rayleigh Fading

- Theoretically, as transmission rate goes to zero, minimum energy per bit (Eb) also goes to zero in Rayleigh fading—we wait until the channel is good enough to send (Verdu).
- In reality, have constraints on latency and bandwidth so that finite Eb always required, and we must expend energy to probe the channel.
- Probing energy will dominate for fast fading channels—better off using standard diversity techniques.